Basic Logic Handout 00

Set Basics

- Notation for standard sets of numbers: $\mathbf{C}, \mathbf{R}, \mathbf{Q}, \mathbf{Z}, \mathbf{N}$
- Standard operators on sets: $\in, \cup, \cap, \subseteq, \notin$
- "Set Builder" notation: $\{$Universal set $\mid$ defining restriction$\}$

1. **Example:** The set of even integers: $\{n \in \mathbf{Z} \mid n = 2k \text{ and } k \in \mathbf{Z}\}$
2. **Example:** The set of Real-valued functions whose domain is the set of Real numbers and whose graph passes through the point $(2,5)$: $\{f : \mathbf{R} \longrightarrow \mathbf{R} \mid f(2) = 5\}$.

Logical Operators and their Truth Tables

- p, q, r, etc represent mathematical statements that are either True or False but not both.

1. **Not:** (negation): $\tilde{}$ is defined by

p	$\tilde{}p$
T	F
F	T

2. **And** (Conjunction): $\wedge$ is defined by

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3. **Or:** (Disjunction): $\vee$ is defined by

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4. **Conditional** (Implication): $\Longrightarrow$ is defined by

p	q	$p \Longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

5. **Equivalence** (If and only if): $\Longleftrightarrow$ $(\equiv)$ is defined by

p	q	$p \Longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies

1. $p \vee \sim p$
2. $\sim\sim p \Longleftrightarrow p$
3. $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
4. $(p \vee q) \Longleftrightarrow (\sim p) \wedge (\sim q)$
5. $(p \wedge q) \iff (\tilde{}p) \vee (\tilde{}q)$
6. $(p \Longrightarrow q) \Longleftrightarrow (\sim q) \Longrightarrow (\sim p)$ contrapositive
7. $(p \Longrightarrow q) \Longleftrightarrow (\sim p) \vee q$
8. $(P \Rightarrow Q) \Longleftrightarrow ((P \wedge \tilde{}Q) \Rightarrow (R \wedge \tilde{}R))$
9. $((p \Longrightarrow q) \Longrightarrow (r \Longrightarrow s)) \iff ((p \Longrightarrow q) \wedge r) \Longrightarrow s$

Contradictions

1. $p \wedge \sim p$

Quantifiers

Universal: $\forall$ **Example:** $\forall x \in \mathbf{R} \quad x^2 + 1 > 0$ is a true statement

Existential: $\exists$ **Example:** There is an integer solution to $x^2 + 5x + 6 = 0$ is a true statement. $(x = -2)$

Negation of quantifiers $\tilde{}\exists x \ (p(x))$ means $\forall x \ \tilde{}p(x)$

Proof Methods

Direct Proof of $H \Longrightarrow C$ or $H \Longrightarrow C_1 \wedge C_2$

1. Start with the (conjoined) hypotheses of H
2. Use nothing but logical steps See below.
3. Deduce C. (Deduce each of the C_i)